SMALL-SCALE POWER SPECTRUM AND CORRELATIONS IN ΛCDM

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ABSTRACT

Cosmological models with a positive cosmological constant ($\Lambda > 0$) and $\Omega_0 < 1$ have a number of attractive features. A larger Hubble constant H_0, which can be compatible with the recent HST estimate, and a large fraction of baryon density in galaxy clusters make them current favorites. Early galaxy formation also is considered as a welcome feature of these models. But early galaxy formation implies that fluctuations on few megaparsec scales spent more time in the nonlinear regime, as compared with standard Cold Dark Matter (CDM) or Cold + Hot Dark Matter (CHDM) models. As has been known for a long time, this results in excessive clustering on small scales. We show that a typical ΛCDM model with $H_0 = 70$ km/s/Mpc, $\Omega_0 = 0.3$, and cosmological constant Λ such that $\Omega_\Lambda \equiv \Lambda/(3H_0^2) = 1 - \Omega_0$, normalized to COBE on large scales and compatible with the number-density of galaxy clusters, predicts a power spectrum of galaxy clustering in real space which is too high: *at least* twice larger than CfA estimates (Park *et al.* 1994) and 3 times larger than APM estimates (Baugh & Efstathiou 1994) for wavenumbers $k = (0.4 - 1)h/$Mpc. This conclusion holds if we assume either that galaxies trace the dark matter ($\sigma_8 \approx 1.1$ for this model) or just that a region with higher density produces more galaxies than a region with lower density. The only way to reconcile the model with the observed power spectrum $P(k)$ is to assume that regions with high dark matter density produce fewer galaxies than regions with low density. Theoretically this is possible, but it seems very unlikely: X-ray emission from groups and clusters indicates that places with a large density of dark matter produce a large number of galaxies. Since it follows that the low-Ω ΛCDM models are in serious trouble, we discuss which ΛCDM models have the best hope of surviving the confrontation with all available observational data.

Subject headings: cosmology: theory — dark matter — large scale structure of the universe — galaxies: clustering

1. INTRODUCTION

Models with density $\rho_0 = \Omega_0\rho_{\rm crit}$ less then the critical density $\rho_0 < \rho_{\rm crit} = 3H_0^2/8\pi G$ and a cosmological constant $\Lambda \neq 0$ have become popular in recent years for a number of reasons. With an age of the universe $t_0 \geq 13$ Gy, such models allow a large Hubble constant $H_0 = (60-80)$ km/sec/Mpc, which agrees with observational indications for large Hubble constant (e.g. Freedman *et al.* 1994; Riess, Press & Kirshner 1994), although the value of H_0 remains uncertain. These models can naturally explain how galaxy clusters could have a large fraction (20%–30%; White *et al.* 1993; White & Fabian 1995) of mass in baryons without severe contradictions with amount of baryons compatible with primordial nucleosynthesis (Walker *et al.* 1991; Krauss & Kernan 1994; Copi, Schramm, & Turner 1995). (Note however that the latest results on clusters, taking into account the higher masses from gravitational lensing, decreases the cluster baryon problem for $\Omega = 1$ models; see e.g. Squires et al. 1995.) Low-Ω ΛCDM models also cure many other problems of the standard CDM model (e.g. Efstathiou, Sutherland, & Maddox 1990; Bahcall & Cen 1992; Kofman, Gnedin, & Bahcall 1993; Cen, Gnedin, & Ostriker 1993; Gnedin 1995b; Park *et al.* 1994; Croft & Efstathiou 1994).

If the Universe has $\Omega_0 < 1$ with $\Lambda = 0$, the model is not compatible with standard inflation. This means that we do not have an explanation for such fundamental observed properties of the Universe as its homogeneity. Measurements of cosmic microwave background anisotropies by COBE and other CMB experiments put severe constraints on any possible inhomogeneity of the Universe. While rather contrived inflationary schemes do appear to be able to generate universes that look from inside like open models (Sasaki et al. 1993; Bucher, Goldhaber, & Turok 1995; Linde & Mehlumzian 1995), it remains uncertain what spectrum of fluctuations to use for structure formation in such models, and also whether they produce a sufficiently homogeneous universe. Thus, open models would solve some problems, but could open more difficult questions. This is the reason why we consider in the present paper flat cosmological models with cosmological constant Λ: $\Omega_{\rm total} = \Omega + \Omega_\Lambda = 1$. Such models can be compatible with inflation scenario in its most general form. They also give a larger t_0 than models with the same Ω_0 with $\Lambda = 0$.

Dynamics of cosmological models with Λ constant were discussed in many papers (e.g. Lahav *et al.* 1991; Kofman, Gnedin, & Bahcall 1993). Already the first N-body simulations (Davis *et al.* 1985, Gramann 1988) revealed a problem with the model: the correlation function is too steep and too large at small scales. For example, Efstathiou *et al.* (1990) found that the model with $\Omega_0 = 0.20$ nicely matches the APM angular correlation function $w(\theta)$ on large angular scales ($\theta > 1°$), but disagrees by a factor of 3 with APM results on smaller scales. The disagreement was not considered serious because "the models neglect physics that is likely to be important on small scales where $\xi(r) >> 1$".

While it is true that one cannot reliably take into account all physics of galaxy formation, it does not mean that we can manipulate the "galaxies" in models without any restrictions. For example, one of the ways out of the problem would be to assume that places with high dark matter density somehow are less efficient in producing galaxies. Coles (1993) shows that if the dark matter density has a gaussian distribution and the number density of galaxies is any function of local dark matter density ("local bias"), then the correlation function of galaxies cannot be flatter then the correlation function of the dark matter. Because the correlation function of dark matter in the ΛCDM model was already too steep at small scales, we cannot satisfy APM $w(\theta)$ constraints *simultaneously* at large and small scales with any local bias. (So, we can reduce small-scale $w(\theta)$, but this will ruin it on large scales.) Thus, in order to reconcile the model with observations we need to appeal to nonlocal effects as used by Babul & White (1991) on small scales or by Bower *et al.* (1993) on large scales. Those nonlocal effects can result from photoionization of ISM

by UV photons produced by quasars, AGNs, and young galaxies. Another source of non-locality is propagation of shock waves produced by multiple supernovae in active galaxies (Ikeuchi & Ostriker 1986). It should be noted that both effects are not very efficient in suppressing star formation in high density environments. UV radiation heats gas only to few tens of thousands of degrees. This does affect the formation of small galaxies and delays the time of formation of the first stars on large galaxies, but it is difficult to see how can it change a large galaxy. If the gas falls into the gravitational potential of normal-size galaxy with effective temperature of about 10^6K, it does not matter much if it was ionized and preheated. Even if a strong shock is produced by an active galaxy, it is difficult to deliver the shock to a nearby galaxy. Because galaxies are formed in very inhomogeneous environments, a shock produced by one galaxy will have a tendency to damp its energy into local void, not to propagate into a dense area where another galaxy is forming. Numerical hydro+N-body simulations of Gnedin (1995a), which incorporate effects of UV radiation, star formation, and supernovae explosions, do not show any antibias of luminous matter relative to the dark matter.

In this paper we are trying to avoid complicated questions about effects of star formation. We estimate the nonlinear power spectrum of dark matter and reinforce the old conclusion that it is not compatible with observed clustering of galaxies. Thus, the model which has a formal bias parameter $b \sim 1$ must have extra (anti-) bias. Then we put lower limits on possible correlation function of galaxies for any local bias, which assumes that larger dark matter density results in more galaxies. The lower limit on the power spectrum is 2–3 times higher than CfA estimates (Park *et al.* 1994) and 3–4 times higher than APM results (Baugh, & Efstathiou 1994).

2. MODEL AND NORMALIZATION

We have chosen the following model as the first representative of ΛCDM models to simulate: $\Omega_0 = 0.3$, $\Omega_\Lambda = 0.7$, $\Omega_{\mathrm{bar}} = 0.026$, $h = 0.70$. The age of the Universe for the model is $t_{\mathrm{Univ}} = 13.4$Gyr. The normalization of the spectrum of fluctuations for the model corresponds to the quadrupole of cosmic microwave anisotropy $Q_{\mathrm{rms-PS}} = 21.8\mu K$ (Stompor, Gorski, & Banday 1995). This gives the rms mass fluctuation σ_8 for top-hat filter with radius $8h^{-1}$Mpc equal to $\sigma_8 = 1.10$. With this amplitude the model predicts a bulk velocity $V_{50} = 355$km/s for a sphere with $50h^{-1}$Mpc.

Another way of expressing and fixing the normalization is the number of rich galaxy clusters. White, Efstathiou & Frenk (1993) estimate a cluster abundance of about $n_{\mathrm{Cl}} = 4 \times 10^{-6}(h^{-1}\mathrm{Mpc})^{-3}$ for masses exceeding $M_{Cl} \equiv 4.2 \times 10^{14}h^{-1}M_\odot$. Biviano *et al.* (1993) give a slightly higher estimate for the same mass limit: $n_{\mathrm{Cl}} = 6 \times 10^{-6}(h^{-1}\mathrm{Mpc})^{-3}$. The ΛCDM model considered in this paper predicts $n_{\mathrm{Cl}} = 3.9 \times 10^{-6}(h^{-1}\mathrm{Mpc})^{-3}$ for $M > M_{Cl}$, which is close to these results. (For discussion of cluster masses in different models see Borgani *et al.* 1995.) Here we use the Press-Schechter approximation with gaussian filter and $\delta_c = 1.50$. A top-hat filter with $\delta_c = 1.68$ gives very similar results. White, Efstathiou, & Frenk (1993) found that their results of N-body simulations of clusters are quite well matched by the Press-Schechter approximation with these parameters. These results disagree with estimates of the mass function given by Bahcall & Cen (1993), who find $n_{\mathrm{Cl}} = 1.8 \times 10^{-6}(h^{-1}\mathrm{Mpc})^{-3}$, which is 2–3 times lower than numbers given by White, Efstathiou & Frenk and by Biviano *et al.* The number of clusters is extremely sensitive to the amplitude of fluctuations. For example, if instead of $\sigma_8 = 1.10$ we would take $\sigma_8 = 0.80$ (Kofman, Gnedin, & Bahcall (1993), Gnedin (1995b)), the number of clusters with $M > M_{Cl}$ for our model would drop to the level $n_{\mathrm{Cl}} = 4.5 \times 10^{-7}(h^{-1}\mathrm{Mpc})^{-3}$, which makes it unacceptable. This also means that there is no room in the model for effects like gravitational waves or a small tilt of the spectrum to reduce the amplitude at small scales in order to ease the problem with the power spectrum and correlation function: lower

amplitude on 1–10 Mpc scales makes the model inconsistent with the number of galaxy clusters.

3. SIMULATIONS

In order to estimate the power spectrum in the nonlinear regime we ran 6 simulations using standard Cloud-in-Cell Particle-Mesh code (Kates *et al.* 1990, Hockney & Eastwood 1981). Six simulations were run with very different box sizes and resolutions (see Table 1). The simulations were started at $z = 45$ and run till $z = 0$ with constant step in expansion parameter $\Delta a = 0.003$. By comparing models with different resolution we can estimate effects of the resolution on the power spectrum in the nonlinear regime.

In order to check our estimates of the number of clusters in the model we ran a simulation with a much larger box and lower amplitude. In this case the box size was $200h^{-1}\text{Mpc}$, the resolution was $\Delta x = 390h^{-1}\text{kpc}$. The amplitude $\sigma_8 = 0.9$ was chosen above the value $\sigma_8 = 0.75 - 0.8$ used by Kofman, Gnedin & Bahcall (1993), and Gnedin (1995b), but below our amplitude $\sigma_8 = 1.1$. We found that the number density of "clusters" with mass larger than $M = 4.2 \times 10^{14}h^{-1}M_\odot$ within Abell radius $r_A = 1.5h^{-1}\text{Mpc}$ is $n_{\text{Cl}} = 1.6 \times 10^{-6}(h^{-1}\text{Mpc})^{-3}$. Just as expected, it is well below Biviano *et al.* (1993) and White *et al.* (1993) estimates.

4. RESULTS

4.1 Power Spectrum

Figure 1 presents the power spectrum of dark matter for our ΛCDM model. The bottom panel shows results of different simulations. On small scales ($k > 1h\text{Mpc}^{-1}$) results show definite convergence: better resolution leads to higher power spectrum, but the difference between simulations gets smaller for better and better resolution. On scales $0.2h\text{Mpc}^{-1} < k < 1h\text{Mpc}^{-1}$ results are fluctuating because of cosmic variance (small number of large structures in a simulation). The top panel shows the averaged nonlinear power spectrum. The spectrum was averaged over all simulations in the range $0.1h\text{Mpc}^{-1} < k < 1h\text{Mpc}^{-1}$. For larger wavenumbers we used results from the two simulations with the best resolution. The spectrum matches the linear spectrum at $k \sim 0.2h\text{Mpc}^{-1}$, but goes significantly above it at larger wavenumbers.

In Figure 2 we compare the nonlinear power spectrum with observational results. Dots are results of the Baugh & Efstathiou (1993) for the APM Galaxy survey. Open circles show results for the CfA survey (Park *et al.* 1993). Formal error bars for each of the surveys are smaller than the difference between the results. For the CfA catalog the power spectrum was estimated in red-shift space and then a correction was made for velocity distortions (see Park *et al.* 1993 for details). Because the corrections are large and model dependent (corrections were found by comparing redshift-space and real-space power spectra of dark matter in a large-box, low resolution ΛCDM PM simulation), these CfA results are slightly less reliable then the APM results. The CfA catalog is also more shallow and it is possible that high values of $P(k)$ in CfA are due to a few nearby large structures like the Great Wall. The full curve represents the power spectrum of the dark matter from Figure 1 (top panel). At $k = 0.5h\text{Mpc}^{-1}$ it is 3 times larger than the APM estimate and 1.6 times larger than the CfA value. Note that no simple scale independent bias can help to reconsile this spectrum with observations because its shape is wrong.

We can place stronger constraints on the model by assuming that the number of galaxies produced in some volume is related to the local dark matter density: $n_{\text{gal}} = f(\rho_{dm})$, where f is some function. One can interpret n_{gal} as a probability to produce

a galaxy given the density of dark matter ρ_{dm}. One also can naively expect that f is a monotonically growing function: the larger the density, the bigger is the number of galaxies. This also agrees with the fact that in places where we know there are large amounts of dark matter (as indicated for example by X-ray emission from clusters and groups), there are more galaxies. (In well-studied clusters such as Abell 2218, the galaxies, gas indicated by X-rays, and dark matter indicated by gravitational lensing or other methods all have essentially the same profile; cf. Squires et al. 1995.) As an extreme case, we can assume that $f = constant$: if there is enough mass to produce a galaxy, it does not matter what is the density. This is *not* a reasonable assumption for galaxy formation. We do observe that clusters and groups are the places with higher concentration of galaxies as compared with, say, peripheral parts of clusters or filaments. But this sets a limit on the possible power spectrum of galaxies in this model: if anything, the power spectrum will be larger than our estimate.

Numerically this was realized in the following way. Using our simulations with the best resolution ΛCDM_c and ΛCDM_f we construct the density field. Then density in all cells (cell sizes are given in Table 1) are set to zero if the mass in the cell is less than $5 \times 10^9 M_\odot$. This is too small to produce galaxies for APM or CfA catalogs. Even if we take a mass-to-light ratio 10, the absolute magnitude would be $M_V = -16.7$ — only a tiny fraction of galaxies with this magnitude is in the catalogs. We assume that all cells with higher density could produce galaxies. But following our arguments, all of them are assigned constant density. The value of the constant does not affect the result because we estimate the power spectrum of fluctuations: $n_{\text{gal}}(x)/\langle n_{\text{gal}}\rangle$.

Results for the power spectrum obtained in this way are shown in Figure 2 as the dashed curve (ΛCDM_f simulation) and the dot-dashed curve (ΛCDM_c simulation). Both models agree on large scales, but on smaller scales ($k > 0.5h\text{Mpc}^{-1}$) the higher resolution in ΛCDM_f resulted in a higher power spectrum. Both models indicated that galaxies must be more clustered than the dark matter at least on scales for which we have observational data: $0.1h\text{Mpc}^{-1} < k < 1h\text{Mpc}^{-1}$. Comparison with APM and CfA results indicate contradictions on the level of factor 2–4. A more conventional galaxy identification method (e.g., high peaks) would imply larger discrepancies.

4.2 Correlation Function

Correlation functions provide another way of looking at the same problem. In Figure 3 we plot the real-space correlation function of dark-matter particles ξ_{dm} as a full curve. For comparison we also show the predictions of linear theory (triangles). The dark-matter correlation function is an average of three simulations with best resolution ($\Lambda\text{CDM}_{c,f}$). A small correction was made to take into account waves longer than the box size. If ξ_{sim} is the correlation function in a simulation with the size of the box L, then the corrected correlation function ξ is

$$\xi(r) = \xi_{\text{sim}}(r) + \Delta\xi(r), \qquad (1)$$

where

$$\Delta\xi(r) = \frac{1}{2\pi^2}\int_o^{2\pi/L} P(k)\frac{\sin(kr)}{kr}k^2 dk. \qquad (2)$$

For radii r less than $10h^{-1}\text{Mpc}$ the correction only slightly depends on r. For box size $L = 80h^{-1}\text{Mpc}$ it was $\Delta\xi = 0.145$ at $r = 10h^{-1}\text{Mpc}$ and $\Delta\xi = 0.154$ at $r < 1h^{-1}\text{Mpc}$. For box size $L = 50h^{-1}\text{Mpc}$ the corrections were 0.344 and 0.393 correspondingly. The corrections are not important for radii less than 3–4 Mpc.

On scales larger than $4h^{-1}\text{Mpc}$ the correlation function of the dark-matter is only slightly higher than what is predicted by the linear theory. But on smaller scales the

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difference is soaring: at $1h^{-1}\mathrm{Mpc}$ the non-linear correlation function is 8 times higher than the linear theory prediction. For $0.8h^{-1}\mathrm{Mpc} < r < 4h^{-1}\mathrm{Mpc}$ the correlation function can be approximated by the power law:

$$\xi_{\mathrm{dm}}(r) = \left(\frac{6.5h^{-1}\mathrm{Mpc}}{r}\right)^{2.4}. \tag{3}$$

The slope 2.4 of the correlation function is too steep as compared with the traditional 1.8.

Figure 3 also shows the usual power law approximation $(5h^{-1}\mathrm{Mpc}/r)^{1.8}$ (dot-dashed line) and real-space results of the Stromlo-APM survey (Loveday *et al.* 1995, Figure 2b) (asterisks). The errors of the latter are about 10% for all points (the size of markers on the plot) except for the first data point (smallest r), where it is about 20–30%. It is interesting to note that on scales 1–5 h^{-1} Mpc there is a very good match of the Stromlo-APM correlation function and the correlation function given by the linear theory (i.e., the fourier transform of the linear power spectrum $P(k)$). In other words, in this cosmological model *there is no room for nonlinear effects* , which are essential on $\lesssim 5h^{-1}\mathrm{Mpc}$ scale. In order for the model to survive, galaxies in the model must be severely antibiased with the biasing parameter $b \approx 1/3$ at $r = 1h^{-1}\mathrm{Mpc}$.

In an effort to mimic galaxies in the model, we identified all dark matter halos with $\delta\rho/\rho > 200$ (see below) in our three high-resolution simulations ($\Lambda\mathrm{CDM}_{c,f}$). The correlation function of the halos is shown as the dashed curve in Figure 3. The slope of the correlation function of halos is even steeper than that of the correlation function of the dark matter for $0.5h^{-1}\mathrm{Mpc} < r < 2h^{-1}\mathrm{Mpc}$. Its amplitude at $1h^{-1}\mathrm{Mpc}$ has not changed. There is small anticorrelation (relative to the dark-matter) on large scales $r > 3h^{-1}\mathrm{Mpc}$, which is consistent with linear bias $b_{\mathrm{lin}} = 1/\sigma_8 = 1/1.1$. The agreement with the Stromlo-APM data has not improved on $\approx 1h^{-1}\mathrm{Mpc}$ scale, and is worse on smaller scales.

The halos were identified in the following way:

(1) We started with the density field defined on our 800^3 mesh. All local density maxima above overdensity limit 30 were tagged. This provides a very large list of candidates, which is used to construct a much shorter list of more realistic halos.

(2) We then found positions of density maxima independently of the mesh. For that we placed a sphere of $70h^{-1}\mathrm{kpc}$ radius (our resolution) around each tagged maximum. Centers of mass of all dark matter particles within each sphere were found. Then sphere centers were displaced to the centers of mass and the procedure was iterated until convergence.

(3) After that only halos with maximum overdensity larger than 200 were selected for analysis. The number of selected halos was a factor of ten smaller than the number of tagged maxima. The radius with mean overdensity 200 was found and this radius and mass within this radius were assigned as the radius and mass of the halo.

(4) Because the procedure of finding all neighbors within large radius is very cpu-time consuming, we set a limit of $0.9h^{-1}\mathrm{Mpc}$ on the maximum possible radius of a halo. Fewer than a dozen extremely large halos were affected by the limit. But those few are very important because they are clusters of galaxies. As the result of the limit, peripheral parts of clusters may be underrepresented in our halo "catalogs". The situation is unclear. While because of overmerging the central parts of clusters ($r \lesssim 300h^{-1}\mathrm{kpc}$) is basically structureless with only one halo found by the algorithm, halos are found in very large numbers in peripheral areas of clusters. The typical number of halos with mass larger than $10^{11}M_{\odot}$ within $0.9h^{-1}\mathrm{Mpc}$ radius for a cluster is about 100. It might be too small

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as compared with real clusters. This also indicates that we might be missing some halos even outside the radius. For example, we might be missing halos which already have fallen through the cluster and were destroyed by its tidal field. Because the destruction of halos depends on many details (rate of infall, age of clusters, trajectories of halos, densities of halos and clusters), it is difficult to estimate how important this effect is outside the $0.9h^{-1}$Mpc radius.

(5) The destruction of halos in groups and clusters of galaxies ("overmerging") significantly affects the correlation function of halos. Note that destruction is not just due to the lack of resolution (Moore, Katz, & Lake 1995). It happens because dark matter halos, which move in a cluster, have densities smaller than the density of the halo at the center of the cluster. Any halo that comes close to the center is distroyed by the tidal force. We expect that, in reality, galaxies survive the destruction because gas loses energy and sinks to the center raising the density. But this physics is of course not included in dissipationless simulations. In order to overcome this problem we assume that big halos with mass M above some limit $M_{\rm bu}$ actually represent not a galaxy, but many galaxies. Halos above $M_{\rm bu}$ are broken up into $M/M_{\rm bu}$ "galaxies", which are then randomly distributed inside the big halo in such a way that their number density falls down as r^{-2} from the center. We have chosen $M_{\rm bu} = 7 \times 10^{12}\ h^{-1}M_{\odot}$, but results are not sensitive to the particular value of $M_{\rm bu}$. In order to avoid the problem of assigning different masses to broken halos, we simply weight the contribution of every halo into the correlation function by the mass of the halo. Thus the contribution of a large halo to the correlation function at distances larger than $1h^{-1}$Mpc is not affected at all by the breaking algorithm.

We have tried another, very different approach to the "galaxy" identification. It goes in line with what we did for the power spectrum. We tagged dark matter particles with estimated overdensity at the position of the particle above some limit. The density was constructed using our 800^3 mesh. The correlation functions of those "galaxies" is shown in Figure 4 as a long-dashed curve (overdensity larger than 200), short-dashed curve (overdensity larger than 500), and dotted curve (overdensity larger than 1000). Results represent averages over the same three simulations considered above. For comparison we also show $\xi_{\rm dm}$ (the full curve), Stromlo-APM results (asterisks), and the 1.8 power law (dot-dashed line). On small scales the correlation function rises with rising overdensity limit. Surprisingly, we found no trend with the overdensity on scales larger than $1h^{-1}$Mpc. The curve for overdensity 200 is almost the same (within 20 per cent) as the curve for halos (Figure 3).

Comparison of Figures 3 and 4 shows that results are very insensitive to details of galaxy identification. It seems that none of the simplest and most attractive schemes for the distribution of galaxies in the model can give correlation functions that agree with observations. Galaxies cannot follow the dark matter. Neither the power spectrum (§4.1) nor the correlation function allow this. The simplest biasing models (halos with overdensity above 200 or density above any reasonable threshold) do not work either: the discrepancy on 2–3 Mpc scales can be reduced, but the situation on smaller scale gets even worse. We see three possible solutions for the contradiction between observed and predicted correlation functions at $\sim 1h^{-1}$Mpc scale:

(1) On scales less then $\sim 1h^{-1}$Mpc galaxies are ten times less clustered than dark matter particles. This cannot be achieved by any local bias. In order for the model to succeed, galaxy formation should be significantly suppressed in galaxy clusters. It seems that there is no observational indication of that: the mass-to-light ratio in clusters does not differ much from that in groups. Also the galaxy distribution follows that of the dark matter.

(2) Estimates of the correlation function and the power spectrum are significantly wrong. At $k = 1h$Mpc^{-1} the power spectrum in APM and CfA surveys was underestimated

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by a factor 2–4, and the Stromlo-APM correlation function is ten times too small at $(1-2)h^{-1}$Mpc.

(3) The model predicts clustering at $\sim 1h^{-1}$Mpc scale which is not compatible with that in the real Universe.

The first alternative, while still possible, does not look very attractive. If we assume it, we would need to admit that there is almost no correlation of dark matter density and number density of galaxies in high density areas. We would basically need to "paint" galaxies on small scales. This must be done carefully to avoid any disturbances on large scales, which seem to be in accord with observations, and also to avoid wrecking the hierarchical scaling observed for galaxies (e.g. reduced skewness S_3 and curtosis S_4 are independent of scale R, where $S_n(R) = \langle \xi^{(n)} \rangle_R / \langle_R \xi^{(2)} \rangle^{n-1}$; this is non-trivial since the n-point correlations scale with the bias as $\propto b^n$, thus leading to $S_n \propto b^n/b^{2n-2} = b^{2-n}$) (see e.g. Bonometto et al. 1995 and refs. therein; cf. also Frieman & Gaztanaga 1994). The second alternative also seems unlikely. While errors are always possible, it seems that the APM and Stromlo-APM surveys must have almost no relevance to the real distribution of galaxies in order for the model to be viable. The third possibility seems to be most likely. It does not mean that all variants of ΛCDM model are at fault, but it implies that the most attractive variants with large age of the Universe, large Hubble constant, and relatively large cosmological constant are very difficult to reconcile simultaneously with the observed clustering of galaxies and with the number of galaxy clusters at the same time.

5. DISCUSSION, AND ALTERNATIVE ΛCDM MODELS

It is instructive to compare the $\Omega_0 = 0.3$, $h = 0.7$ ΛCDM model that we have been considering with standard CDM and with CHDM — which is just CDM with the addition of some light neutrinos whose mass totals about 5-7 eV (Klypin et al. 1995, hereafter KNP95; Primack et al. 1995; and refs. therein). At $k = 0.5h$ Mpc^{-1}, Figs. 5 and 6 of KNP95 show that the $\Omega_\nu = 0.3$ CHDM spectrum and that of a biased CDM model with the same $\sigma_8 = 0.67$ are both in good agreement with the values indicated for $P(k)$ by the APM and CfA data, while the CDM spectrum with $\sigma_8 = 1$ is higher by about a factor of two. In the present paper we see that for the ΛCDM model we have simulated, $P(k)$ for the same k is at least as high as CDM, and even higher for larger k. Unless the data is misleading, or some sort of complicated galaxy formation physics leads to a $P(k)$ below our estimates, this is a serious problem for the ΛCDM and standard CDM models.

But the ΛCDM model we have considered here is but one of many. So let us look in h-Ω_0 space for alternatives. In Table 2 we have compared predictions of a number of such models with data on several length scales. In this Table, all the models (except for the ΛCDM one marked with an asterisk) are normalized to the 2-year COBE data as recommended by Stompor, Gorski, and Banday (1995). The first two lines of numbers give our estimates of several observational quantities and the uncertainties in them, from large to small scales. The bulk velocity at $r = 50$ h^{-1} Mpc is derived from the latest POTENT analysis (Dekel 1995 and private communication); the error includes the error from the analysis but not cosmic variance. However, similar constraints come from other data on large scales such as power spectra that may be less affected by cosmic variance since they probe a larger volume of the universe. We have estimated the current number density of clusters (N_{clust}) with $M > 10^{15}h^{-1}M_\odot$ from comparison of data on the cluster temperature function from X-ray observations with hydrodynamic simulations (Bryan et al. 1994) as well as from number counts of clusters (White et al. 1993, Biviano et al. 1993). All recent estimates of the cluster correlation function give fairly large values at

30 h^{-1} Mpc (e.g. Olivier et al. 1993, Klypin & Rhee 1994, and refs. therein); this also suggests that the zero crossing of the correlation function must exceed $\sim 40\ h^{-1}$ Mpc.

The cluster constraint is powerful. Models with Hubble constant h as high as 0.8 cannot have Ω_0 larger than 0.2 and still satisfy the age constraint $t_0 > 13$ Gy, but for such high h and low Ω_0 the COBE normalization leads to too low a cluster density. The cluster density is in good agreement with observations for the $\Omega_0 = 0.3$, $h = 0.7$ ΛCDM model that we have considered in this paper, but as we mentioned above this means that we do not have the freedom to lower the normalization in order to lower the value of the nonlinear $P(k)$. Table 2 shows that the linear $P(k)$ is already fairly high at $k = 0.1h$ Mpc^{-1}, and as Fig. 2 shows, the nonlinear spectrum is considerably higher than the linear one, especially at larger k where the conflict with the APM and CfA data becomes significant. Of course, as we lower h and raise Ω_0, we approach standard CDM for which the cluster abundance is far too high with COBE normalization. Thus for ΛCDM models with intermediate Ω_0 and h correspondingly low to satisfy the t_0 constraint, the cluster abundance goes up. Consider for example $\Omega_0 = 0.5$ and $h = 0.6$. With COBE normalization the cluster abundance is about five times too high, which means that there is room to lower the spectrum and perhaps resolve the $P(k)$ and $\xi(r)$ discrepancies on small scales without exceeding the COBE upper limit on large scales. For example, the line in the table marked ΛCDM* represents a biased version of the same model, with the spectrum simply lowered by a constant factor. (This could in principle be due to gravity waves contributing on the COBE scale, although we admit that we are unaware of an inflationary model with a large gravity wave contribution but no tilt. Indeed, recent work on inflationary cosmologies in the context of supersymmetric models (Dine, Randall, and Thomas 1995; Ross and Sarkar 1995) suggests that inflation will occur at a lower energy scale so that gravity wave production will not be cosmologically significant. Alternatively, we could tilt the model, i.e. assume a primordial spectrum $P_p(k) \approx A k_p^n$ with primordial spectral index n_p less than the Zel'dovich value of unity. Or we could consider a ΛCHDM model, i.e. ΛCDM with a small contribution to the density from light neutrinos. The case shown in Table 2 corresponds to a single neutrino species with a mass of 2.44 eV, consistent with the preliminary results from the LSND experiment (Athanassopoulos et al. 1995). All of these alternatives lower the spectrum significantly and would no doubt result in $P(k)$ in better agreement with the data than the COBE-normalized $\Omega_0 = 0.3$, $h = 0.7$ ΛCDM model considered here. The same is true of the CHDM models, represented in Table 2 by two variants of the two-neutrino model with $\Omega_\nu = 0.20$. The first of these is the Harrison-Zel'dovich ($n = 1$) version considered in Primack et al. 1995, but with the new COBE normalization; now the cluster density is too high. But as the next line shows, a small tilt ($n = 0.96$) with gravity waves corresponding to a $V \propto \phi^2$ chaotic inflationary model gives an excellent fit to all the data considered in the Table. All these models would differ in their predictions for other measurable quantities such as small-angle CMB anisotropies in the first Doppler peak region, production of early objects such as damped Lyman α systems, and small-scale velocities. We regard the further investigation of all these models as an important project for the future.

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FIGURE CAPTIONS

Figure 1. Power spectrum of dark matter for ΛCDM model. The bottom panel shows results of different simulations. On small scales ($k > 1h\mathrm{Mpc}^{-1}$) results show definite convergence: better resolution leads to higher power spectrum, but the difference between simulations gets smaller for better and better resolution. On scales $0.2h\mathrm{Mpc}^{-1} < k < 1h\mathrm{Mpc}^{-1}$ results are fluctuating because of cosmic variance (small number of large structures in a simulation). The top panel shows the averaged power spectrum.

Figure 2. Comparison of the nonlinear power spectrum in ΛCDM model with observational results. Dots are results for the APM Galaxy survey. Open circles show results for the CfA survey. Formal error bars for each of the surveys are smaller than the difference between the results. The full curve represents the power spectrum of the dark matter from Figure 1 (top panel).

Figure 3. Real-space correlation function of the dark-matter (full curve) in ΛCDM model. For comparison we also show predictions of the linear theory (triangles). The usual power law approximation $(5h^{-1}\mathrm{Mpc}/r)^{1.8}$ is shown as the dot-dashed line. Real-space results of the Stromlo-APM survey are presented by asterisks. Correlation function of halos with overdensity more than 200 is shown by the dashed curve.

Figure 4. The correlation function of dark matter with overdensity more than 200 (log-dashed curve), 500 (short-dashed curve), and 1000 (dotted curve). Other curves and symbols in this Figure are the same as in Figure 3. Note that while the correlation function on small scales ($< 1h^{-1}\mathrm{Mpc}$) significantly depends on the density threshold, it does not depend on the threshold on large scales.